DISCONTINOUS GALERKIN METHOD FOR NONLINEAR DISPERSIVE EQUATIONS

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We develop a local discontinuous Galerkin method for solving fully nonlinear dispersive equations that have compactly supported traveling waves solutions, the so-called ``compactons". When a L^2 or L^4 norm stability could be obtained, we design the fluxes at the cell boundary to ensure provable stabilities bounds in these norms for the numerical solution. In other cases, we analyze the corresponding linearized equation and generalize the linearly stable scheme to the nonlinear equations. The numerical simulations we present verify the desired properties of the methods, including their expected order of accuracy. This class of nonlinear dispersive equations may not have the property of integrability, our numerical experiments illustrate the elasticity between collisions. In particular, we demonstrate the potential advantages of using discontinuous Galerkin methods over pseudo-spectral methods in situations where discontinuous fronts are combined with rapid oscillations.

References

[1] J. Yan and C.-W. Shu, "A Local Discontinuous Galerkin Method for KdV Type Equations," *SIAM J. Numer. Anal. ,* no. 2, (2002), pp. 769-791.